Coupling between multiple acoustic domains through elastic structures: Rapid solutions using Fast Multipole BEM

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Modeling coupling between multiple acoustic domains interacting through elastic structures is important in many applications. For example, prediction of the noise transmitted from the exterior to the vehicle interior through window panels requires a fully coupled analysis of the exterior and interior acoustic fields along with the window panel vibration. Coupled acoustic-structure interaction problems have been historically studied; however, analytical solutions are available only for ideal geometries such as sphere, cube, and plate. For complicated geometries given arbitrary boundaries, numerical methods must be utilized. Despite significant work done in the recent past, there are still many challenges. For instance, how do we extend these numerical methods to models of larger dimensions, higher frequency regimes, and multiple acoustic domains?

In this paper, a fully coupled formulation that can efficiently handle multiple acoustic domains and elastic structures will be proposed. We use the Fast Multipole boundary element method (BEM) to model acoustic domains and the finite element method (FEM) to model the elastic structures; the Fast Multipole BEM is very attractive and a faster method to analyze ultra large acoustic models that are valid at mid to high frequency regimes. Further, we employ *invacuo* structural modes to reduce the FEM model. The coupled system of equations for the acoustic fields in multiple domains and the structure vibration are simultaneously solved. Examples of real-life applications such as noise transmission through vehicle window panels, etc., are discussed. The proposed methods could be extended to other applications that require a fully coupled analysis such as exhaust pipe shell radiation, acoustic pressure loading on aerospace vehicles, noise transmission between coupled rooms, etc.

1 INTRODUCTION

For many vibrating structures the acoustic loading is weak, and hence the structural vibration motions are not affected. However for some thin elastic structures, structures with closed acoustic spaces, or structures submerged in dense fluids, the acoustic loading is sufficiently significant to influence the structure motion. This may be viewed as a feedback

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coupling path between the acoustic pressure and the structure vibration. Thus the governing equations for the structure vibrations and acoustic wave equations must be simultaneously solved. Moreover in some applications noise generated in one acoustic domain is transmitted to another acoustic domain through an elastic structure, introducing feedback coupling between the two acoustic domains. In such cases it is important to model both acoustic domains and the coupling with the interacting structure. For example, the sound transmission through a thin impervious wall between two rooms can only be modeled by considering the coupling between the two acoustic domains and the wall.

Coupled acoustic-structure interaction problems have been historically studied; however, closed form analytical solutions are available only for ideal geometries such as sphere, cylinder, rectangular box, plates, etc[1-2]. For more complicated geometries and arbitrary boundary conditions numerical methods must be utilized. Due to its intrinsic advantages, Boundary Element Method (BEM) is widely used for modeling acoustic problems, whereas Finite Element Method (FEM) is quite popular for modeling structure vibrations. In this paper we present a coupled BEM-FEM formulation to model strongly coupled vibro-acoustic problems. The structural FEM model is further simplified with the use of *invacuo* modes. The BEM calculations are accelerated by Fast Multipole Method (FMM). The FMM enables fast matrix-vector computations, and when employed with iterative methods, produces rapid solutions. In recent years the emergence of FMM-BEM for acoustic analysis has dramatically shifted the utility of BEM for small low frequency models to ultra large models valid for mid-high frequency regimes.

2 COUPLED BOUNDARY ELEMENT AND FINITE ELEMENT FORMULATION

The boundary element method is used to formulate the acoustics problem and finite element method is used to formulate the structure vibrations. For coupled systems, the unknown acoustic pressure on the surface in the BEM model acts as a loading on the structure in the FEM model and the unknown structure vibration in the FEM model acts as the excitation for the BEM model. Thus we obtain a fully coupled formulation as described below.

2.1 Single Acoustic Domain Coupled to a Structure

In this formulation a single acoustic domain is assumed to be coupled with an elastic structure. The Kirchhoff-Helmholtz integral equation over the boundary surface S of a radiating structure is given by,

$$\boldsymbol{C}(\boldsymbol{P})\,\boldsymbol{p}(\boldsymbol{P})+\int_{S}\boldsymbol{p}(\boldsymbol{Q})\frac{\partial \boldsymbol{G}(\boldsymbol{P},\boldsymbol{Q})}{\partial \boldsymbol{n}_{q}}\boldsymbol{dS}(\boldsymbol{Q})=\int_{S}\frac{\partial \boldsymbol{p}(\boldsymbol{P})}{\partial \boldsymbol{n}}\boldsymbol{G}(\boldsymbol{P},\boldsymbol{Q})\boldsymbol{dS}(\boldsymbol{Q})+\boldsymbol{p}^{inc} \tag{1}$$

where $p(P)$ and $\frac{\partial \boldsymbol{p}(\boldsymbol{P})}{\partial \boldsymbol{n}}=\boldsymbol{j}\rho\omega\boldsymbol{v}_{n}$ are the pressure and pressure gradient at point P on the surface, ρ is the density of the medium, ω is the frequency of analysis in rad/sec, $\boldsymbol{v}_{n}$ is the normal velocity, and $\boldsymbol{j}=\sqrt{(-1)}$, $\boldsymbol{C}(\boldsymbol{P})$ is proportional to the solid angle at P, $\boldsymbol{G}(\boldsymbol{P},\boldsymbol{Q})$ is the free-space Greens function, and $\boldsymbol{p}^{inc}$ is the incident pressure on the boundary due to acoustic sources. The boundary conditions applied to the surface S can be any combination of velocity (S_v), pressure (S_p), impedance (S_z), and acoustic-structure coupling (S_c), as shown in Fig. 1. However, for simplicity we discuss the case with only the coupled acoustic-structural boundary condition ($S=S_c$). Note that we follow the $e^{-j\omega t}$ convention throughout this paper, but it is dropped for the sake of brevity.

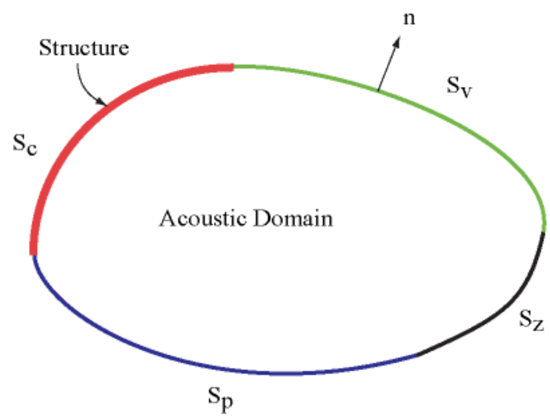


Fig. 1 – Coupled formulation with a two-way acoustic-structural coupling (S_c) and other possible boundary conditions.

Discretization of the surface of the radiator into boundary elements and assembling equations for all collocation points, the integral Eqn. (1) produces a system of equations

$$\mathbf{Ap} = \mathbf{r} + \mathbf{p}^{\mathbf{inc}} \tag{2}$$

where $\mathbf{A}$ and $\mathbf{r}$ are the left-hand side matrix, and right-hand side vectors, $\mathbf{p}$ is the vector of unknown pressures at collocation points, and $\mathbf{p}^{\mathbf{inc}}$ is the vector with incident pressures due to acoustic sources. The right-hand side vector $\mathbf{r}$ depends on the structure velocity, $\mathbf{v}$, and is further reduced using modal superposition $\mathbf{v} = -\boldsymbol{j}\omega\mathbf{\Phi}\eta$,

$$\mathbf{r} = -\boldsymbol{j}\rho\omega\mathbf{Rv} = -\rho\omega^2\mathbf{R\Phi}\eta \tag{3}$$

The rectangular matrix $\mathbf{R}$ relates collocation points to the structure nodes; $\mathbf{\Phi}$ is the structure *invacuo* modal matrix; η is vector of modal coefficients. For coupled acoustic-structural problems η is unknown and needs to be solved simultaneously with $\mathbf{p}$. The final system of equations representing the BEM acoustic domain is

$$\mathbf{Ap} = -\rho\omega^2\mathbf{R\Phi}\eta + \mathbf{p}^{\mathbf{inc}} \tag{4}$$

Finite element method is used to formulate structure vibrations. The system of equations modeling structure vibrations in the frequency domain is given by

$$\left\{-\omega^2\mathbf{M} - \boldsymbol{j}\omega\mathbf{C} + \mathbf{K}\right\}\mathbf{u} = \mathbf{f}^{\mathbf{a}} + \mathbf{f}^{\mathbf{s}} \tag{5}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are finite element mass, damping, and stiffness matrices, respectively, $\mathbf{u} = \mathbf{\Phi}\eta$ is the displacement vector, $\mathbf{f^a}$ is the acoustic loading, and $\mathbf{f^s}$ is the external loading on the structure. We assume that the *invacuo* structure modes $\mathbf{\Phi}$ are ortho-normalized with respect to the mass matrix. Thus,

$$\mathbf{\Phi^T M \Phi} = \mathbf{I} \qquad \mathbf{\Phi^T K \Phi} = \begin{bmatrix} \ddots & & \\ & \omega_{ni}^2 & \\ & & \ddots \end{bmatrix} = \mathbf{\Omega^2} \qquad \mathbf{\Phi^T C \Phi} = \begin{bmatrix} \ddots & & \\ & 2\varsigma_i \omega_{ni} & \\ & & \ddots \end{bmatrix} = \mathbf{\Lambda} \tag{6}$$

Equation (5) reduces to

$$\mathbf{Z}\eta = \mathbf{\Phi^T f^a} + \mathbf{\Phi^T f^s} \tag{7}$$

where

$$\mathbf{Z} = \left\{ -\omega^2 \mathbf{I} - \boldsymbol{j}\omega\mathbf{\Lambda} + \mathbf{\Omega^2} \right\}$$

$\mathbf{Z}$ is a diagonal matrix and hence easy to invert. The acoustic loading $\mathbf{f^a}$ is related to acoustic pressures $\mathbf{p}$ on the surface by a matrix $\mathbf{Q}$.

$$\mathbf{f^a} = \mathbf{Qp} \tag{8}$$

To solve the coupled acoustic-structural problem, we combine Eqn. (4) and Eqn. (7) to form the fully coupled system of equations.

$$\begin{bmatrix} \mathbf{A} & \rho\omega^2 \mathbf{R\Phi} \\ -\mathbf{\Phi^T Q} & \mathbf{Z} \end{bmatrix} \begin{Bmatrix} \mathbf{p} \\ \boldsymbol{\eta} \end{Bmatrix} = \begin{Bmatrix} \mathbf{p^{inc}} \\ \mathbf{\Phi^T f^s} \end{Bmatrix} \tag{9}$$

Typically, iterative methods are used to solve Eqn. (9) for the pressure (**p)** and modal coefficients (η) simultaneously. Due to the scaling difference between the acoustic and elastic domains the left-hand side matrix is not well conditioned and results in poor convergence. Hence, we eliminate η from Eqn. (9) to obtain system of equations in only one variable $\mathbf{p}$ as

$$\left\{ \mathbf{A} + \rho\omega^2 \mathbf{R}\Phi\mathbf{Z^{-1}}\Phi^T \mathbf{Q} \right\} \mathbf{p} = -\rho\omega^2 \mathbf{R}\Phi\mathbf{Z^{-1}}\Phi^{\mathbf{T}} \mathbf{f^s} + \mathbf{p^{inc}} \tag{10}$$

Equation (10) could be interpreted as a boundary element formulation similar to Eqn. (2) with additional terms for coupling with the structure. Iterative methods in conjunction with FMM are used to efficiently solve for $\mathbf{p}$. The modal coefficient vector η is then computed from Eqn. (7).

2.2 Accelerating Computations using Fast Multipole Method

The fast multipole method offers an efficient way of computing matrix-vector products for matrices having special structure. All BEM matrices from the above formulation have this special structure due to the presence of Greens function or its derivatives and hence can take advantage of FMM. FMM, when used in conjunction with iterative methods from the Krylov family, results in rapid solutions. A multi level fast multipole method (MLFMM) as described by Gunda and Vijayakar[3] is used to accelerate BEM computations.

In Eqn. (10), **A** is a matrix generated from the BEM formulation and hence the matrix-vector product **Ap** could be sped up using FMM straight away. The coupled term in the left-hand side involves many matrix-matrix products and looks like a potential bottle neck. However, many of these computations are easy to perform except the matrix-vector product involving **R** which is fully populated and frequency dependent. For example, the matrix-matrix product involving $\mathbf{\Phi}$ and **Q** is frequency invariant and is required to be evaluated only once per analysis. More over the matrix **Z** is diagonal and the matrix **Q** is sparsely populated. To accelerate computations involving **R** matrix we take advantage of its structure, which contains Greens function, and employ FMM.

2.3 Multiple Acoustic Domains Coupled through a Structure

The formulation for the single acoustic domain can be extended to consider multiple acoustic domains coupled to the same structure. Figure 2 shows two acoustic domains enclosed by surfaces S_1+S_c and S_2+S_c. The two domains are coupled to the structure at S_c. Assuming that the two domains are coupled only through the structure and not by any other means; the system of equations can be written as

$$\begin{bmatrix} \mathbf{A_1} & 0 & \rho_1\omega^2\mathbf{R_1\Phi} \\ 0 & \mathbf{A_2} & \rho_2\omega^2\mathbf{R_2\Phi} \\ -\mathbf{\Phi^T Q_1} & -\mathbf{\Phi^T Q_2} & \mathbf{Z} \end{bmatrix} \begin{Bmatrix} \mathbf{p_1} \\ \mathbf{p_2} \\ \boldsymbol{\eta} \end{Bmatrix} = \begin{Bmatrix} \mathbf{p_1^{inc}} \\ \mathbf{p_2^{inc}} \\ \mathbf{\Phi^T f^s} \end{Bmatrix} \tag{9}$$

where subscripts 1 and 2 represent different acoustic domains. Rewriting Eqn. (9) by eliminating η we obtain

$$\begin{bmatrix} \mathbf{A_1}+\rho_1\omega^2\mathbf{R_1}\Phi\mathbf{Z}^{-1}\Phi^T\mathbf{Q}_1 & \rho_1\omega^2\mathbf{R}_1\Phi\mathbf{Z}^{-1}\Phi^T\mathbf{Q_2} \\ \rho_2\omega^2\mathbf{R_2}\Phi\mathbf{Z}^{-1}\Phi^T\mathbf{Q_1} & \mathbf{A_2}+\rho_2\omega^2\mathbf{R}_2\Phi\mathbf{Z}^{-1}\Phi^T\mathbf{Q_2} \end{bmatrix} \begin{Bmatrix} \mathbf{p_1} \\ \mathbf{p_2} \end{Bmatrix} = \begin{Bmatrix} -\rho_1\omega^2\mathbf{R_1}\Phi\mathbf{Z}^{-1}\Phi^T\mathbf{f}^s+\mathbf{p_1^{inc}} \\ -\rho_2\omega^2\mathbf{R_2}\Phi\mathbf{Z}^{-1}\Phi^T\mathbf{f}^s+\mathbf{p_2^{inc}} \end{Bmatrix} \tag{10}$$

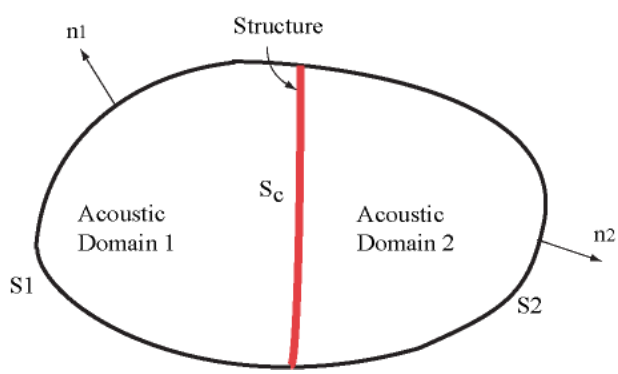
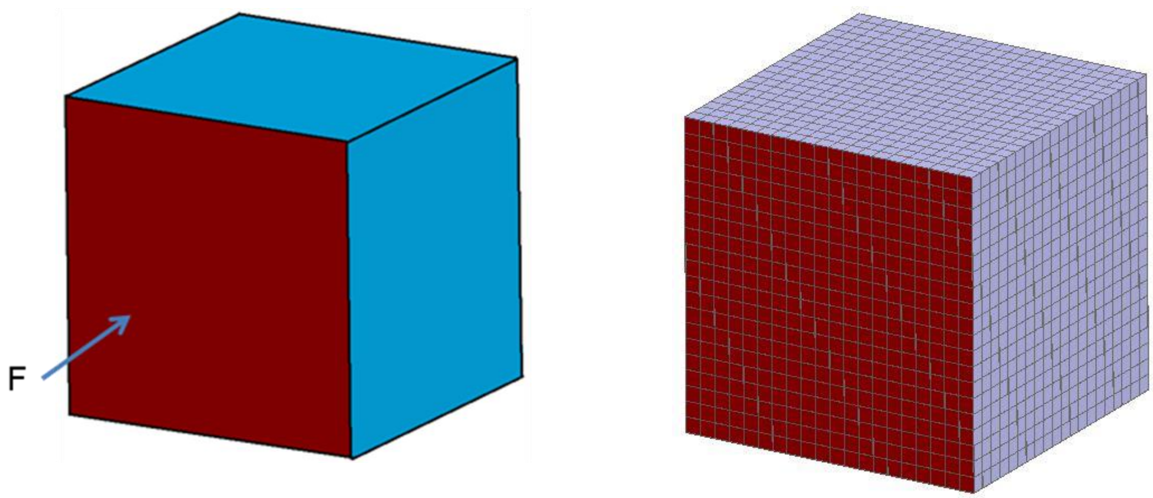


Fig. 2 – Two acoustic domains coupled to the same vibrating structure.

3 CASE STUDY

3.1 Validation: Rectangular panel backed by a closed cavity

In this example, the coupling between an elastic rectangular panel and the acoustic cavity backing it is analyzed. Our formulation is validated by comparing with the analytical series solution derived by Pretlove[2]. A schematic of the model with the boundary element mesh is shown in Fig. 3. The cavity has dimensions of 1 m x 1 m x 1 m and is backed by the elastic panel on one side and rigid walls on others. The panel has a thickness of 1 cm and is assumed to be made of steel with Young's modulus E = 210 GPa, density ρ_s = 7900 kg/m^3, and Poisson's ratio ν = 0.3. The fluid medium inside the cavity is assumed to be water with a sound speed c = 1481 m/s and density ρ_w = 1000 kg/m^3. As the analytical series solution derived by Pretlove assumes only coupling between the elastic panel and the cavity, we neglect the exterior domain and model only the interior acoustic domain for this study.



(a) *(b)*

Fig. 3 – Elastic panel (in red) backed by cavity; (a) schematic with driving force shown, and (b) boundary element mesh.

Invacuo structural modes of the panel for simply supported boundary conditions are computed using finite element software and are used to model the structure in our coupled formulation. The first six modal frequencies from FEA are compared to analytical expressions in Table 1. The corresponding *invacuo* structural mode shapes are shown in Fig. 4.

A unit force is applied on the panel at (0.2, 0.3) and the driving point displacement from our formulation is compared with the analytical solution in Fig. 5. Excellent agreement is found between results from our formulation and the analytical solution. The discrepancy in the magnitudes of the response at resonant frequencies is due to the fact that our formulation requires structural damping for proper convergence and the analytical solution does not include any damping. We note that the structural resonant frequencies for the coupled system are different from the *invacuo* structural modal frequencies due to the cavity loading. For modes with nonzero average flux the cavity has an added stiffness effect and hence shifts the corresponding resonances to higher frequencies, whereas for modes with zero average flux the cavity has an added mass effect and hence shifts the resonances to lower frequencies.

Table 1 – Invacuo natural frequencies of a plate with simple supports.

Mode (*r*,*s*)	Analytical (*Hz*)	FEA (*Hz*)
(1,1)	49.0	49.0
(1,2)	122.5	122.4
(2,1)	122.5	122.4
(2,2)	196.1	195.8
(1,3)	245.1	244.7
(3,1)	245.1	244.7

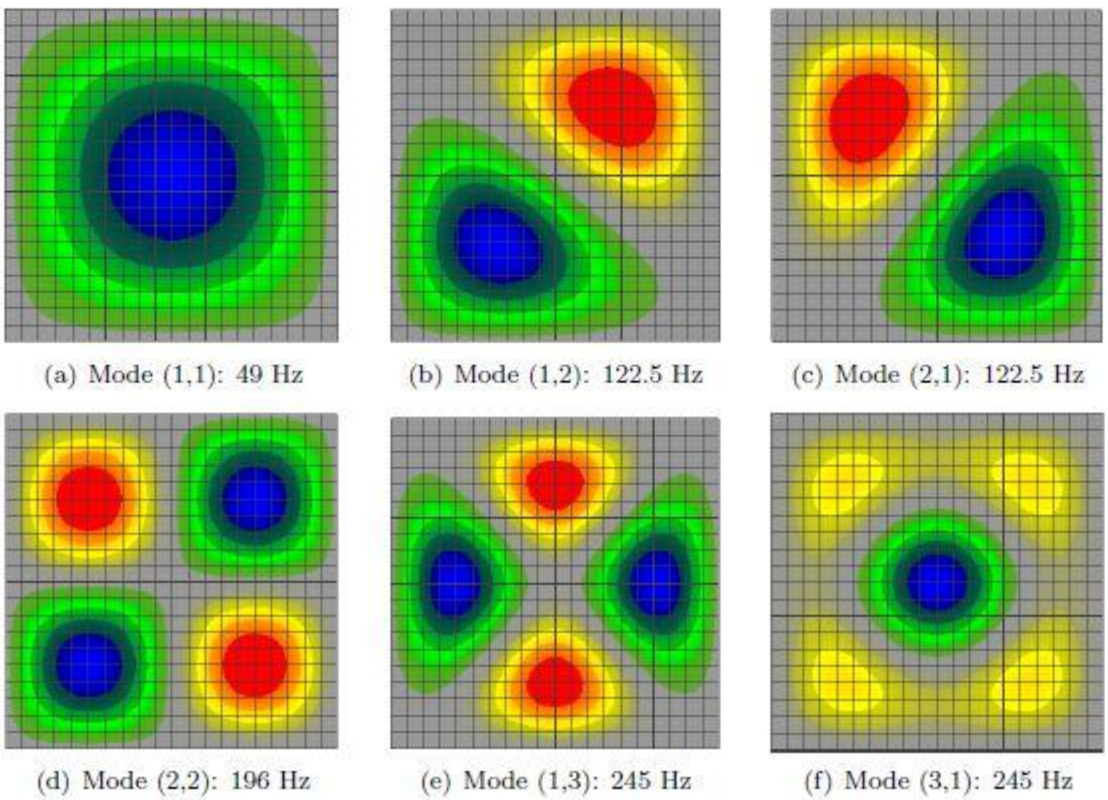


Fig. 4 – Invacuo mode shapes for the first six vibration modes of a plate with simple supports.

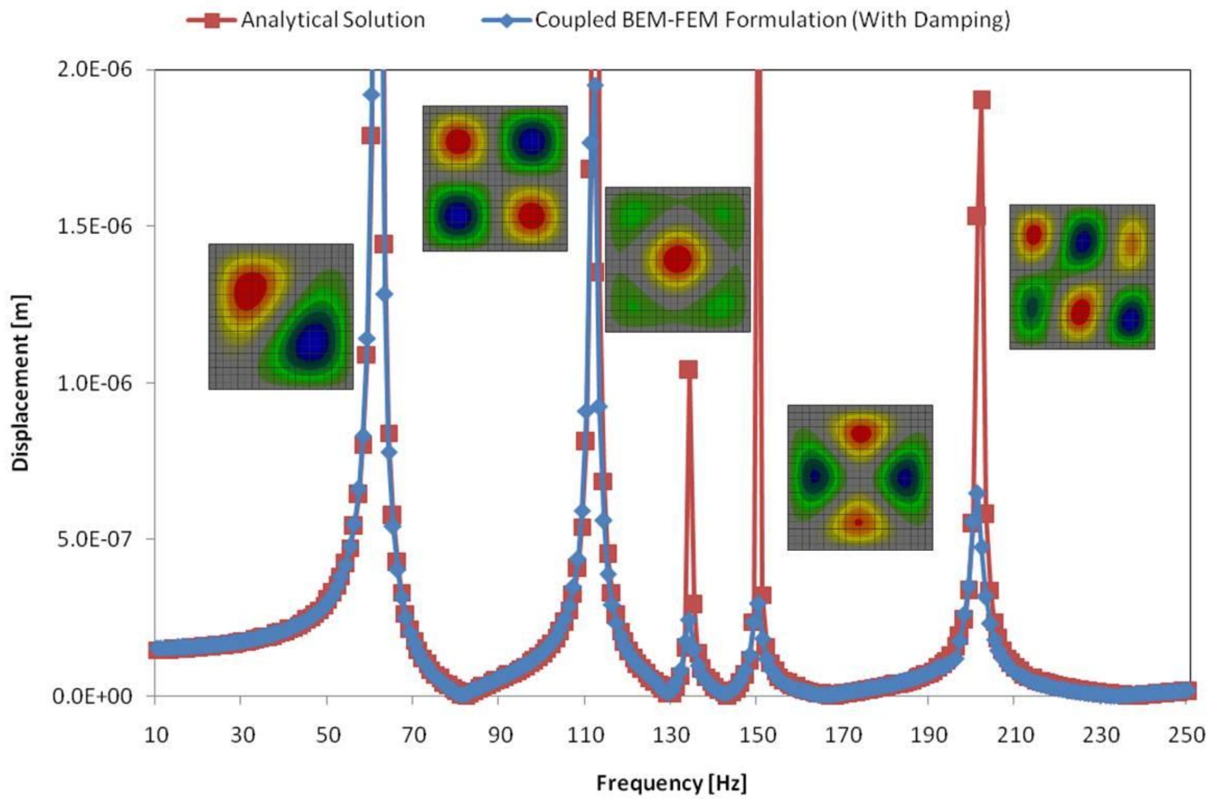


Fig. 5 – Driving point frequency response of the elastic panel coupled to the cavity. Structural modes corresponding to each resonant peak are also shown. Damping is added to the structure in the Coupled BEM-FEM Formulation to ensure the solution convergence but no damping is considered in the Analytical Solution.

3.2 Exterior noise transmission to car interior through a window panel

In this example, we examine the exterior air borne noise transmission to the interior of a car through a window panel. The exterior noise excites the window panel that is coupled to the interior acoustic cavity. Hence a fully coupled model involving both the interior and exterior acoustic domains along with the window panel structure is necessary to analyze this system. Figure 6 shows a detailed model of the car exterior and the interior along with the window panel. The car exterior model has dimensions of approximately 4.7 m x 2 m x 1.3 m and the corresponding BE mesh has around 600,000 elements. The car interior including interior walls, seats, and driver's head is modeled with around 70,000 boundary elements. For this study, only the driver's side window panel is assumed to be flexible and hence coupled to the exterior and interior acoustic domains. The rest of the car is assumed to be rigid. The fluid medium is assumed to be air with a sound speed c = 343 m/s and density ρ_a = 1.21 kg/m^3.

A finite element mesh of the window panel along with boundary element meshes for both the exterior and interior acoustic domains is shown in Fig. 7. The window panel has a thickness of 0.5 cm and is assumed to be made of glass of Young's modulus E = 72 GPa, Shear modulus G = 29.8 GPa, and density ρ_s = 2520 kg/m^3. Only the bottom portion of the panel is rigidly fixed (constrained in all directions), and the rest of the panel is left free. *Invacuo* structural modes are computed from finite element software and are used to model the structure in the coupled BEM-

FEM formulation. This detailed model helps us understand various vibro-acoustic properties of the coupled system. For example, we can use this model to evaluate transmission loss of the window panel, or examine the effects of different acoustic liners (impedance material) on the transmitted sound, etc. Figure 8 shows the transmission loss of the window panel. An external plane wave source located 1 m away from the center of the window panel is used as the excitation and the sound pressure is measured at the driver's ear for this evaluation.

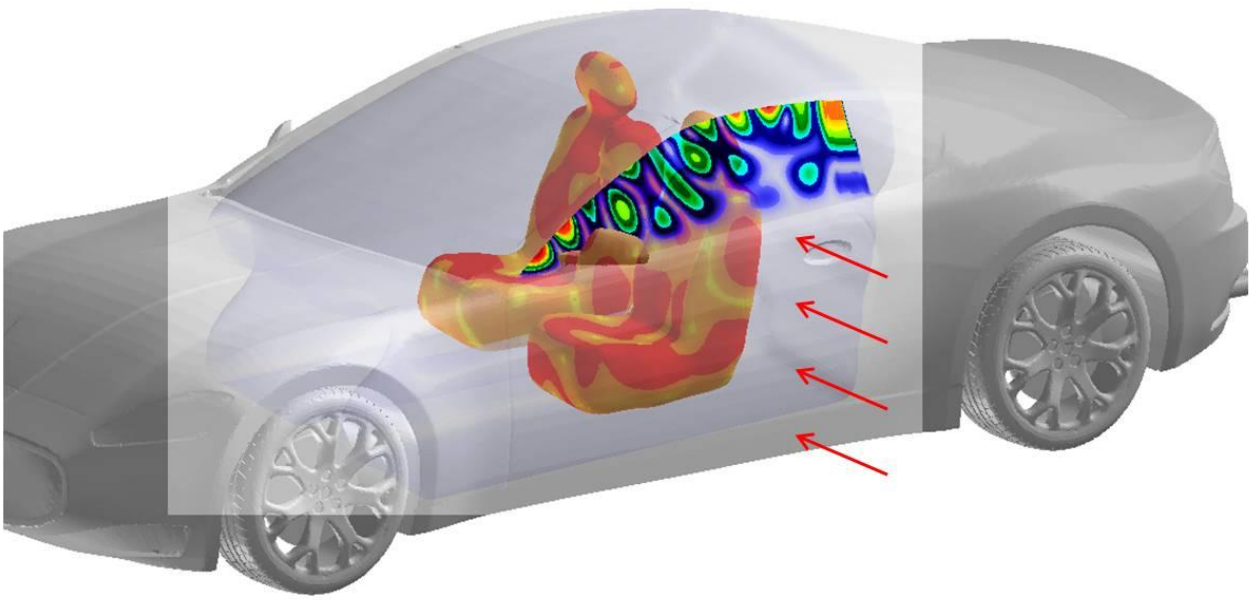
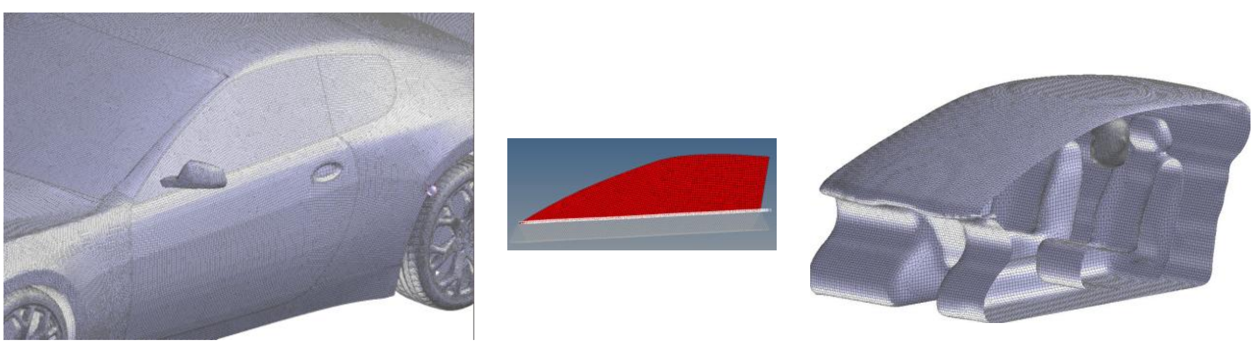


Fig. 6 – Noise transmission from an external acoustic source to the car interior through a window panel at 1000 Hz. Window panel surface velocities are superimposed on the sound pressure levels in the interior.



(a) Surface mesh used to model the exterior BE acoustic domain

(b) FE shell mesh used to model the window panel

(c) Surface mesh used to model the interior BE acoustic domain

Fig. 7 – Different meshes used to model the fully coupled multi-domain acoustic-structural interaction problem.

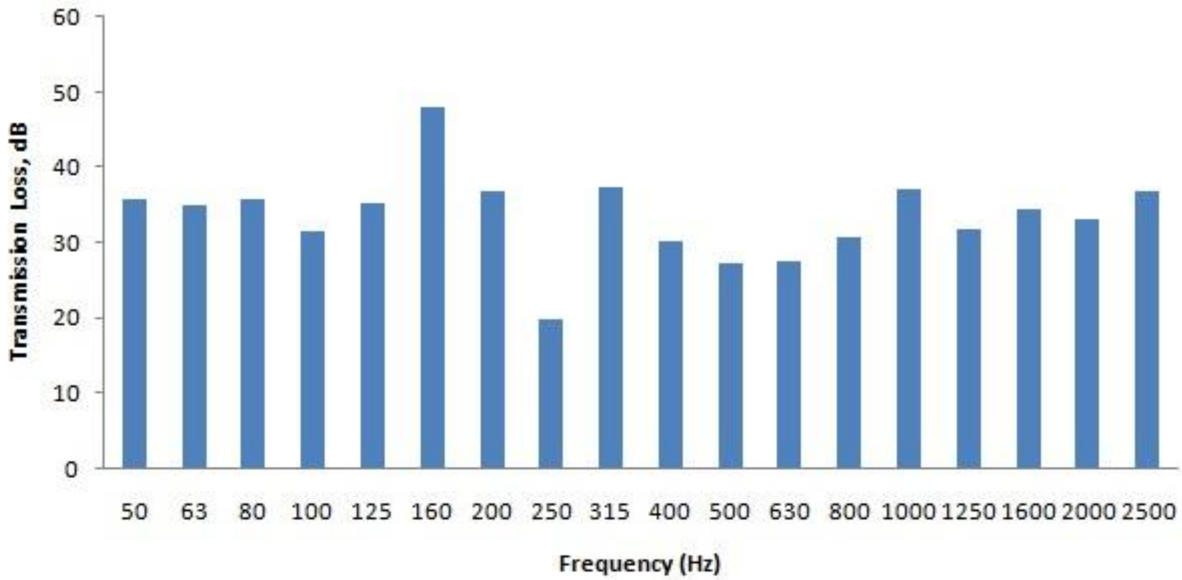


Fig. 8 – Transmission loss of the window panel.

4 SUMMARY AND CONCLUSIONS

A fully coupled multi-domain BEM-FEM formulation to model multiple acoustic domains interacting through elastic structures is presented. All applicable computations are accelerated using FMM. The structural model using FEM is further reduced using *invacuo* structural modes. Different case studies are presented. The proposed methods are very general and could be extended to model other systems that require a fully coupled analysis such as exhaust pipe shell radiation, acoustic loading on aerospace vehicles, sound transmission through walls, etc.

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